

Empirical analysis of collective human behavior for extraordinary events in blogosphere

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To uncover underlying mechanism of collective human dynamics, we survey more than 1.8 billion blog entries and observe the statistical properties of word appearances. We focus on words that show dynamic growth and decay with a tendency to diverge on a certain day. After careful pretreatment and fitting method, we found power laws generally approximate the functional forms of growth and decay with exponents around -1. We also observe news words whose frequency increase suddenly and decay following power laws. In order to explain these dynamics, we propose a simple model of posting blogs involving a keyword, and its validity is checked directly from the data. The model suggests that bloggers are not only responding to the latest number of blogs but also suffering psychological pressure from the divergence day. Our empirical results can be used for predicting the number of blogs in advance and for estimating the period to return to the normal fluctuation level.

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1. INTRODUCTION

Collective motions are recognized as a typical example of self-organization caused by neighbor interactions. Thereby it has been widely investigated by physics community [1, 2]. In particular, collective behavior in human society has attracted considerable interest in the last decade. Because developments in information technology have enabled the storage of large volumes of high-frequency human activity data. For instance, detecting bubbles in stock exchange activities [3], modeling dealer behavior using real data in the foreign exchange markets [4], and the empirical analysis of consumer behavior in supermarkets and convenience stores using purchase history and point of sales (POS) data [5, 6]. Human activity data that is collected from the web, for example, YouTube videos and the social network service Facebook, are analyzed to not only explain basic individual human behavior but also elucidate hidden network structures in the society [7, 8].

Here we also use the data from the web to uncover non-trivial mechanism of collective human activities. Because word frequency on the web is expected to immediately reflect the real social mood, it has attracted increasing attention among many academic and industrial researchers. In fact, they are stored electronically and analyzed widely. For example, the Library of Congress in the United States which is the largest

library in the world has been archiving the entire public tweet of Twitter, a micro-blogging system since 2007, (<http://blog.twitter.com/2010/04/tweet-preservation.html>).

A blog is a type of website that is maintained by an individual with entries displayed chronologically with time stamps. The term “blog” originated from the combination of “web” and “log,” and was popularized around the year 2000 when free blog services began to be provided by internet service companies. A “blogger,” who is an owner of a blog site, can easily upload his/her “entries” any time, and readers can easily post comments on the blog page. This interactive quality has contributed to the success of blogs; they are now widely used as basic social communication tools. The collective community of blogs is often called the “blogosphere.” There exists a relation between the blogosphere and various scientific fields such as statistical physics, engineering, sociology, linguistics, and psychology.

In this study, we analyze the keyword appearance rate in blogs in which the functional forms of growth and decay around the peak are approximated by power laws. For earthquake research, the frequency of aftershocks is reported to decrease following a power law of time after the main shock; this is known as Omori’s Law [9]. Similar power laws have been established in other fields of human activity. For example, a power law can describe a decrease in online book sales with an exponent that depends on endogenous or exogenous shocks [10]. Relaxation in audience number for online movies can also be described by power laws with various values of exponents that reflect the quality of the content [7]. Alfi et al. found that growth in conference registration numbers

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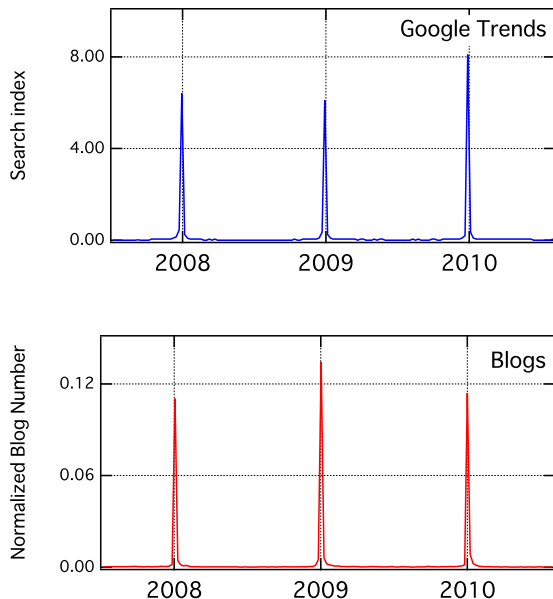


FIG. 1: (Color online) Temporal change of the word frequency of “April fool” per week. The results are from Google Trends, which is targeted worldwide and our blog data “Kuchikomi@kakaricho,” which is targeted only in Japan. The number of blogs is normalized by the whole number.

is also approximated by the power law diverging at the deadline [11].

In Sec. 2, we describe the analyzed data and Japanese blogs. In Sec. 3, we introduce our pretreatment procedures and peaked words. In Sec. 4, we focus on the time evolution of these peaked words and prove that they grow and decay with power laws. To reproduce power laws, we introduce a simple model of posting blogs in Sec. 5. In Sec. 6, we discuss the predictability of our model from the standpoint of application, and the final section is devoted to conclusions.

2. DATA DESCRIPTION

The data analyzed in this study was obtained from the blogosphere written in Japanese over a period of four years, from November 1st 2006 to October 31st 2010. According to the technical report by the internet search engine company Technorati (<http://technorati.com>), that tracked more than 70 million blogs worldwide in 2007, the share of Japanese blogs is 37%, the largest among all languages. Although we only analyze the Japanese blogosphere, we show an example in which the dynamic properties in Japanese and English are considerably similar. Figure 1 shows the temporal change of the frequency of the English “April Fool” observed by Google Trends (<http://www.google.com/trends>) surveyed world-

wide compared to the number of blog entries containing the corresponding Japanese. In both cases, we confirm that there is a clear peak on the week including April fools’ day.

In blogosphere research, it is important to note the existence of spam blogs. They are automatically generated blogs in which the same words are repeated multiple times, mainly for the purpose of advertising. As the share of spams in the Japanese blogosphere is said to be 40%, it is important to exclude spams from the data. We used a new internet service called “Kuchikomi@kakaricho” (<http://kakaricho.jp>) to collect the data. This service provides an application programming interface (API) that counts the number of entries in which a given target word appeared in a given period by using a search engine technology with a spam filter. There are three levels of spam filtering and we apply the middle level, which is known to remove most of the spams while keeping most of the human blogs untouched. The API counts the number of entries in the blogs such that if one entry includes the target word multiple times, the word is counted only once. @

The API started crawling the blogosphere on November 1st 2006 and covered major blog service providers. It covers more than 1.8 billion blog entries in 15 million blogs accounting for 90% of the Japanese blogosphere.

For analysis of Japanese we introduced a pretreatment to separate Japanese words that are not separated by spaces. Here we use the commonly used Japanese morphological analyzer “MeCab” (<http://mecab.sourceforge.net/>) to individually separate words according to a dictionary. By adding words to its dictionary, this software can treat multi-word phrases such as “April Fool” as one word, “April-Fool”. Most of the words used in this study are already listed in the software’s dictionary as one word, except names of people.

3. PEAKED WORDS

In the blogosphere, there are special words whose frequency grows or decays around a peak day such as “April Fool” with the peak on April 1st. In the following discussion, we denote these words as “peaked words” and analyze their functional forms of growth and decay.

3.1. Pretreatment

We first apply the following pretreatment to the data to exclude both trivial circadian human activity patterns and systematic noises.

Time-Shift In blogosphere, although a day starts at 00:00:00, there are many bloggers who are active at midnight. Therefore, we examine the complete circadian activity pattern and introduce a type of

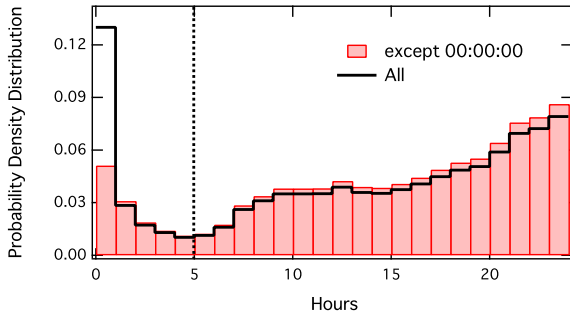


FIG. 2: (Color online) Probability density distribution of circadian activity of blog posted by 10000 bloggers. Solid line is calculated from all entries and the red bar is from the entries excepted that have time stamp of 00:00:00. In both, 4:00 is the smallest ratio in a day.

correction pretreatment for our daily data. For this purpose, we randomly chose the data of 10,000 bloggers with the details of their activities time stamped in seconds. By counting the number of entries posted at every hour, a circadian activity pattern is plotted in Fig. 2. The solid line shows the 24-hour-activity pattern obtained directly from the data. However, we discovered there are a certain number of blog entries with time stamps that are exactly 00:00:00. We consider this time stamp to be caused by an artificial systematic spec or error, and we exclude this data from the statistics when capturing the circadian pattern. The red bars in Fig. 2 show the revised circadian activity pattern. Using a 24-hours clock, we find that blogging activity is lowest around 4:00, and thus we consider the start of a day at 5:00 to be reasonable. Because the share of activity in the interval between 0:00 and 5:00 is approximately 10% of the complete activity of a day, we can correct the daily number of blog entries including the j -th target word at the t -th day $\tilde{x}_j(t)$ by the following equation

$$x'_j(t) = w\tilde{x}_j(t) + (1-w)\tilde{x}_j(t+1), \quad (1)$$

where the weight is set as $w = 0.9$. With this modification, we can determine the time-shifted time series. In Fig. 3, open circles show the original daily data in which $w = 1.0$ in Eq. (1), and colored circles show time-shifted data in which $w = 0.9$. The time-shifted data shows a more symmetric pattern than the original data. We also apply this procedure to determine the time series of the total number of blog entries per day $x'(t)$. To clarify the effect of this time-shift procedure, we also show results without this time-shift procedure in Appendix A.

Normalization There are non-uniform and non-stationary properties in the total number of entries per day [12]. For example, there was a sudden drop

in February 2007 that was caused by search engine software's system maintenance. In order to reduce the systematic fluctuations caused by such non-uniform properties, we apply the following normalization procedure. There is already a method to separate internal and external noises [13], which simply deducts external factor depending on its ratio of the total traffic. They assume that each traffic $x'_j(t)$ in a small component $j \in N$ is consisted of the total traffic $x'(t)$ without overlap, where $\sum_{j=1}^N x'_j(t) = x'(t)$. Here, we simply divide $x'_j(t)$ by the total number, $x'(t)$. The normalized number of entries for the j -th word on the t -th day is defined by $x_j(t) = x'_j(t) \frac{\langle x' \rangle}{x'(t)}$, where $\langle x' \rangle$ denotes the mean value of $x'(t)$ that is averaged over the entire observation period. This normalized quantity is proportional to the probability that a blog contains the j -th word on the t -th day, and it is not necessarily an integer.

By introducing this normalization, the fluctuations caused by the aforementioned non-uniform properties can be reduced. In this study, we measure the word frequency using this normalization procedure.

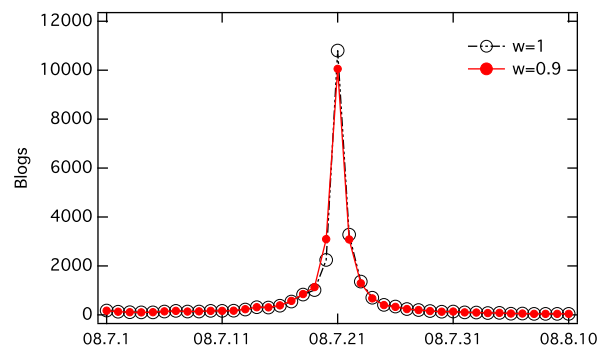


FIG. 3: (Color online) Typical example of time series of peaked word “Marine Day” in 2008. $w = 1$ corresponds to no revision and $w = 0.9$ corresponds to modified time series introduced in Sec. 3.1. Because of the circadian effect, the data of the day after the peak is always higher than that before the peak without modification.

3.2. Word Selection

We determine candidates for peaked words in the following three categories.

Event We selected the names of 14 public holidays and 16 major annual events in Japan. The appearance for these words grows and decays around the date of the event. In addition, these are words affiliated

with an event, such as “Santa Claus” for “Christmas” and we can observe similar growth and decay behavior for those words. However, in this analysis we neglected such affiliated words.

Date We selected dates such as “9th May,” resulting in 365 words. There are many blog entries that announce some special day, e.g., birthday and festival. Growth and decay of these words always show a clear peak at the date.

News A word such as “earthquake” occurs suddenly right after the occurrence of the event and the word appearance rate generally decays slowly. In order to observe the functional form of such decay after a significant event, we selected names of the places impacted by earthquakes. We also selected 33 names of famous people who died suddenly. In addition, we included the names of the Japanese scientists who received a Nobel Prize during our observation period.

4. DYNAMICS OF PEAKED WORDS

We call the slopes before the peak day fore-slopes and those after the peak day after-slopes, and we examine both. As the study on power law statistics has not been cultivated enough, there is no standard method to check the validity of a power law approximation for given data [14]. A promising statistical test for power law distributions is based on Kolmogorov-Smirnov statistics [15], here, we generalize the method to time series.

4.1. Method

We define the number of days in each slope by the number of consecutive days in which the word frequency is larger than the median value \bar{x}_j from the peak. The median value is estimated throughout the entire observation period. Then we approximate the functional form of the slopes using two models, a power law and an exponential law.

$$x_j(t) - \bar{x}_j = A_j |t_c - t|^{-\alpha_j} \quad (2)$$

$$x_j(t) - \bar{x}_j = B_j \exp(-\beta_j |t_c - t|) \quad (3)$$

The parameters of these models, α_j , A_j , β_j , and B_j , are determined by the least squares method. The fitting region is $[t_c \pm 1, t_c \pm n]$ where n is the number of days in slope. Then we apply the Kolmogorov-Smirnov goodness of fit test, for choosing the better model. It was originally used as a statistical test for distributions. Here, we apply it for evaluation of the statistical fitness of the functional form of the time series. For both models we calculate the KS statistic D , representing the deviation, is defined as

$$D = \max_{t \in [t_c \pm 1, t_c \pm n]} |X_j^{(empirical)}(t) - X_j^{(model)}(t)|, \quad (4)$$

where $X_j^{(empirical)}(t)$ is the cumulative number of empirical value which is counted from the data, and $X_j^{(model)}(t)$ is the cumulative number which is calculated from the model. In both cases, numbers are normalized by $X_j(t_c \pm 1)$. By comparing the values of D for both models, the power law model is accepted if the D -value for the power law is smaller. In the case that the power law is accepted, we check the validity of the model as introduced in [14]. First we generate 1000 synthetic data set. One data set contains n data points. Synthetic data points are generated randomly following the normal distribution with the mean value is best estimated from the model $x_j^{(model)}(t)$ and standard deviation is $\sigma(x_j^{(model)}(t))$ as follows

$$\sigma(\langle x_j \rangle) \simeq \sqrt{\langle x_j \rangle (1 + a^2 \langle x_j \rangle)}, \quad (5)$$

where $a = \frac{\sqrt{\langle X^2 \rangle_c}}{\langle X \rangle} = 0.08$ is a constant parameter characterizing the fluctuation in the number of all bloggers which is determined independently of the word (see Appendix B for theoretical derivation of Eq. (5)). For each synthetic time series, we compare its D -value with that of the empirical one. We count the number of cases in which the D -value for the synthetic time series is larger. If the number of such cases are less than 100 out of the 1000 synthetic samples, we accept the power law model as $q = 0.1$. Contrary to the ordinary sense of p -value, the power law hypothesis is considered to be valid for larger q . Thus if the q is close to 1, then the difference between empirical data and the model can be attributed to statistical fluctuation alone and we accept power law hypothesis. If the q is smaller than 0.1, we reject power law hypothesis. We change the border of the fitting region n from 5 days to maximum slope length. The value of power exponent, α_j , is given by the value for the case with the largest n .

4.2. Results

Figure 4 is a typical result of data fitting for the word “Marine Day” in 2008 with log-log scale, as shown in Fig. 3 with linear scale. For all cases of power law fitting, the distribution of the estimated power exponents are shown in Fig. 5 and summarized in Tab. I.

The absolute value of the power exponents of the after-slopes is larger than that of the fore-slopes in 58% of the 65 samples for Event, and 80.6% of the 603 samples for Date. For Date, we confirm significant difference between fore-slopes and after-slopes by t -test with p -value $< 2 \times 10^{-16}$ while it is rejected with p -value = 0.80 for Event. The number of days of the after-slopes is larger than that of the fore-slopes in 55% of the 65 samples for Event, and 65.8% of the 603 samples for Date. For Date, we confirm significant difference between fore-slopes and after-slopes by KS-test with p -value $< 2 \times 10^{-16}$ while it is rejected with p -value = 0.22 for Event.

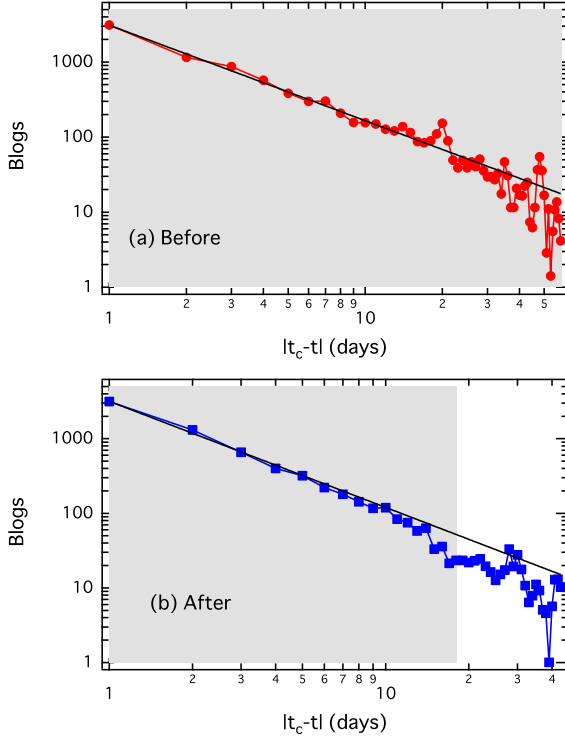


FIG. 4: (Color online) Examples of data fitting by power laws of “Marine Day” in 2008 for fore-slope (a) and after-slope (b) plotted in log-log scale. For fore-slope, models are fitted by Eq. (2) with $\alpha_j = 1.27$ and $A_j = 3100$ ($q = 0.239$, $n = 58$). For after-slope, $\alpha_j = 1.42$ and $A_j = 3171$ ($q = 0.108$, $n = 18$). The shaded area shows the interval n in which the power law model is accepted.

In the case of the news words, there is no fore-slope and we cannot compare the values of the exponents before and after the peak. The absolute values of the exponent after the peak tend to be estimated as smaller for high impact news because of the effect of sequential broadcasts after the news. For example, in the case of the sudden death of the world famous entertainer Michael Jackson, which marked the peak day, there was a funeral service after a few days and a memorial CD released after a few weeks. Both can be regarded as aftershocks that remind us of the main news. Because of such repetition, the keyword appearance rate after the peak day is enhanced, the decay of the word appearance becomes slower, and the power exponent tends to take a smaller value.

4.3. An extreme case “Tsunami”

The power law decay per day of the word “Tsunami” in the Japanese blogosphere is shown in Fig. 6(a). The peak day was March 12th 2011, the day after the quake with 142617 posts or 12.6 % of all blog posts in raw data. After pretreatment of time-shift and normalization, the

TABLE I: Mean values of power exponent α_j with standard deviations and medians of slope days n .

		α_j	n (days)	# samples
Event	Before	1.40 ± 0.38	10	83
	After	1.44 ± 0.28	16	91
Date	Before	0.79 ± 0.38	9	776
	After	1.11 ± 0.16	21	1229
News	After	1.09 ± 0.45	10	21
All	Before	0.85 ± 0.30	9	859
	After	1.13 ± 0.21	20	1341

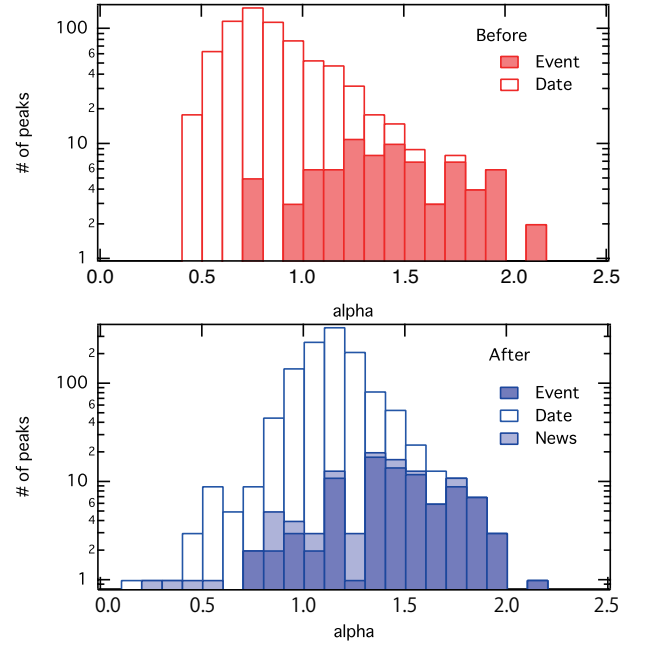


FIG. 5: (Color online) Distribution of power exponent α_j of the fore-slopes (a) and the after-slopes (b). Mean value of α_j of fore-slope is 0.85 ± 0.30 and after-slope is 1.13 ± 0.21 .

estimated power exponent α_j is 0.67 with $A_j = 61788$ ($n=50$) using Eq. (2). It is expected to take approximately 8623 days (~ 23.4 years) to return to the normal fluctuation level if we simply broaden power law function. The normal fluctuation level was 140 appearances per day, estimated from the data one month before the quake. Although most of the news words decay in approximately 10 days, the case of “Tsunami” is a rare exception because the number of entries is still ten times higher than before the peak even for a year after the quake.

Twitter also shows similar power law behavior even though the time resolution is different. Figure 6(b) shows the number of tweets measured per hour that include “Tsunami” that is calculated based on 1397783 tweets. We believe that this type of power law reflects the robust-

ness of the empirically observed dynamics of collective human behavior.

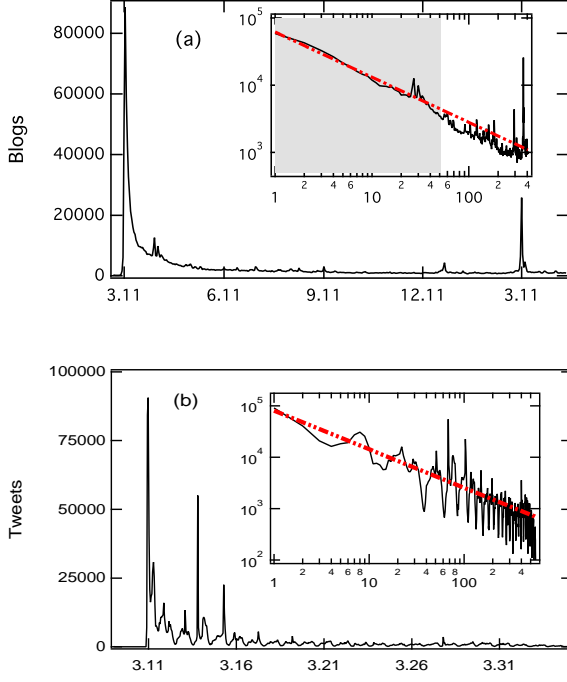


FIG. 6: (Color online) Decay of “Tsunami” observed for blogs (a) and tweets (b). Horizontal time step size is per day for blogs and per hour for tweets. (Inset) Log-log plots of the time series. Red dashed lines show the slope of power law with the exponent $\alpha_j = 0.67$ for blogs and $\alpha_j = 0.75$ for tweets.

5. THE MODEL

In this section, we propose a simple dynamic model to describe the typical power law growth and decay of frequency of blogs with peaked words. There is already a simple model to describe people’s universal behavior before a deadline by assuming pressure inversely proportional to the remaining time [11]. As this simple model can describe only the special case $\alpha = 1$, a kind of utility function that includes the tendency to postpone the action is introduced to describe the general case. Here, we introduce another approach to describe the general case. We use the following two assumptions for the number changes of blogs $\Delta x_j(t) = x_j(t+1) - x_j(t)$, increments for fore-slope and decrements for after-slope.

1. The pressure from the peak day t_c works inversely proportional to the time, $1/|t_c - t|$ [11].
2. The number of changes $\Delta x_j(t)$ is proportional to the number of blogs $x_j(t)$.

We can write these two assumptions into mathematical form in continuous case as we assume $\Delta x_j(t) \simeq \frac{dx_j(t)}{dt}$. The time evolution of blogs for the fore-slope is given as

$$\frac{dx_j(t)}{dt} = \alpha_j^{(fore)} \cdot \frac{x_j(t)}{(t_c - t)} + f(t), \quad (6)$$

where $f(t)$ is an independent noise with zero mean. The value $\alpha_j^{(fore)} > 0$ is a proportionality factor that describes the effect of the above-mentioned two assumptions. Similarly, the decrement of the after-slope is given as

$$\frac{dx_j(t)}{dt} = -\alpha_j^{(after)} \cdot \frac{x_j(t)}{(t - t_c)} + f(t), \quad (7)$$

where $\alpha_j^{(after)} > 0$ is also a proportionality factor that describes the effect of the two assumptions. Because we know that blogs decrease after t_c , we use a negative value in the Eq. (7). It confirms that Eq. (6) and Eq. (7) derive the solution of Eq. (2) without the noise term. Thus, $x_j(t) \propto (t_c - t)^{-\alpha_j^{(fore)}}$ for fore-slopes and $x_j(t) \propto (t - t_c)^{-\alpha_j^{(after)}}$ for after-slopes. In the case that there is no pressure from the peak day t_c , blog dynamics follow Eq. (3) of the exponential law. We rewrite Eqs. (6) and (7) into the following discrete form without noise term $f(t)$ and check the validity from the data in Fig. 7.

$$\frac{|\Delta x_j(t)|}{x_j(t)} = \alpha_j \cdot \frac{1}{|t - t_c|} \quad (8)$$

Note that t_c is not necessarily an integer since the divergence point is expected to exist in single day time period. This result suggests that bloggers not only respond to the number of blogs but they also get pressure from the peak day t_c .

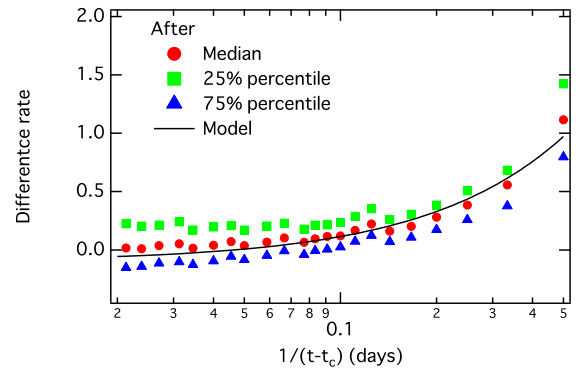


FIG. 7: (Color online) Relationship between difference rates $|\Delta x_j(t)|/x_j(t)$ and the absolute inverse number of time from the peak calculated by Eq. (8) for after-slopes, which summarized 1341 samples shown in Tab. I. Solid line shows model with empirically observed exponent $\alpha_j = 1.13$.

6. PREDICABILITY OF FREQUENCY

As an application of this study, we explore the possibility of estimating the word frequency in the near future. In Fig. 8, we show an example of prediction of blog frequency “Marine Day” in 2008. In this case, we already have the information about the peak days to be July 21st 2008; thus, we can fix the divergence point t_c . From the data, we find that the slope period starts on April 28th, 85 days before t_c , as the normalized frequency continuously exceeds the median value from this day. In Fig. 8(a), the case of prediction for 20 days before the divergence point using 65 data points with Eq. (2) is shown by the red line. In Fig. 8(b), the case of prediction for 5 days before the peak day is shown. The prediction error becomes smaller for shorter prediction period as expected. Note that a small difference in estimation of the exponent α_j makes a big difference near the peak; thus, the number of data points plays an important role in its accuracy.

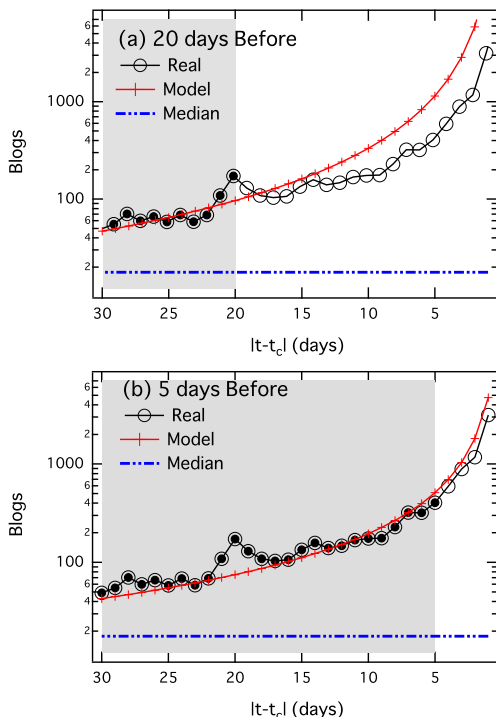


FIG. 8: (Color online) Typical examples of prediction for (a) 20 days and (b) 5 days before the peak day for “Marine Day” in 2008 in semi-logarithmic scale. Red line indicates the prediction line, blue dashed line indicates median $\bar{x}_j = 17$, and solid line shows the real values. Open circles indicate the future values and colored circles are the known values used for prediction. The estimation is started 85 days before t_c . Estimated values are $\alpha_j = 1.78$ and $A_j = 20222$ for 20 days before the peak day and $\alpha_j = 1.38$ and $A_j = 4725$ for 5 days before t_c .

7. CONCLUSIONS

By analyzing a large database of Japanese blogs, we showed that the functional forms of growth and decay of word appearance that peaked on a certain day are generally approximated by power laws with the exponents around -1. The values of the power exponents depend on the category of words such as Event, Date, and News. In the case of Event and Date, clarification of asymmetry in the power exponents of the fore-slope and after-slope is an interesting subject for future research on collective human behavior. In the case of News, the power law can be observed only after the peak, and its power exponent depends on its impact. In the case of significant news such as the March 11th earthquake in 2011, the absolute value of the power exponent is clearly smaller than 1.

We also checked the validity of a simple model that indicates that bloggers change their probability of posting proportional to the number of blogs and inversely proportional to the time interval from the peak. The model suggests that bloggers do not only respond to the number of blogs but also they spontaneously get pressure from a peak day. In addition, these power functions can be observed also in Twitter, and it suggests that these power law behaviors are universal in social phenomena. An agent-based mathematical model will be used reproduce these empirical properties of blogger activity in the near future [16].

Acknowledgments

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Appendix A: Case without time-shift

We show the results without the time-shift; thus Eq. (1) with $w = 1$. Figure 9 shows a typical example of data fitting without time-shift for the word “Marine Day” as mentioned in Sec. 4.2. There is no major change in power exponent α_j for fore-slope and after-slope. However, for the value of intercept A_j , we can find major deviation especially for fore-slope ($A_j = 3171$ with time-shift and $A_j = 2273$ without time-shift). In Fig. 10 and Table II, we summarize the whole samples.

Appendix B: Modified Random Diffusion Model

We introduce a modified random diffusion model, which is used in Eq. (5). The random diffusion model was originally introduced to describe diffusion properties

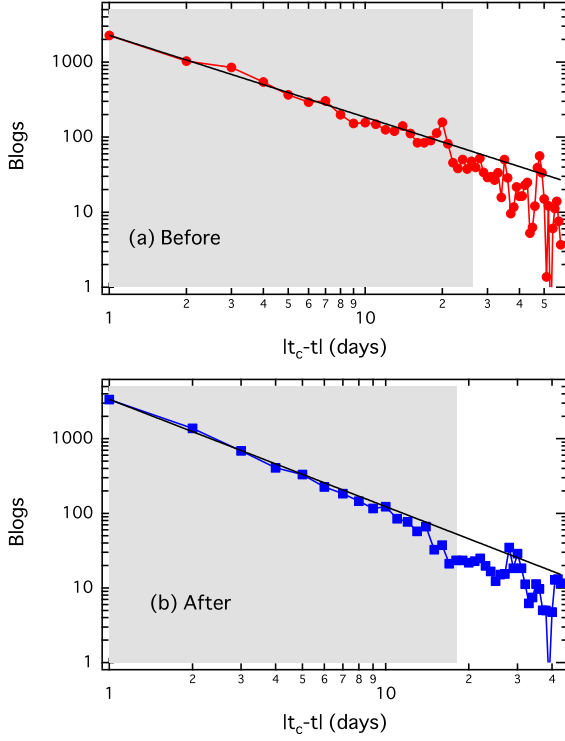


FIG. 9: (Color online) Examples of data fitting without time-shift pretreatment by power laws of “Marine Day” in 2008 for fore-slope (a) and after-slope (b) plotted in log-log scale. For fore-slope, models are fitted by Eq. (2) with $\alpha_j = 1.10$ and $A_j = 2273$ ($q = 0.103$, $n = 26$). For after-slope, $\alpha_j = 1.44$ and $A_j = 3369$ ($q = 0.104$, $n = 18$). The shaded area shows the interval in which the power law model is accepted with the p -value less than 0.1.

TABLE II: Mean values of power exponent α_j with standard deviations and medians of slope days n in case without time-shift procedure which introduced in Sec. 3.1.

		α_j	n (days)	# samples
Event	Before	1.21 ± 0.38	15.5	80
	After	1.48 ± 0.28	16	85
Date	Before	0.64 ± 0.25	11.5	418
	After	1.14 ± 0.18	22	1176
News	After	1.21 ± 0.35	11	18
All	Before	0.73 ± 0.35	12	498
	After	1.16 ± 0.21	21	1279

of random walkers on a given network [17, 18], and two of the authors (Y.S. and M. T.) have modified the model to be applicable to the fluctuations in word appearance in the blogosphere [19]. In our modified random diffusion model, we assume that there are two states, active and non-active for each blogger, and the number of active bloggers fluctuates randomly each day. Each active

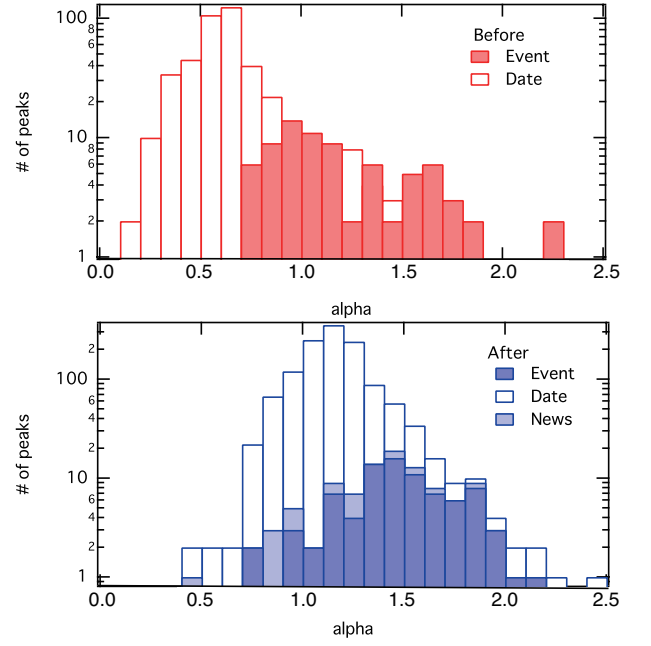


FIG. 10: (Color online) Distribution of power exponent α_j of fore-slopes (a) and after-slopes (b) without time-shift. Mean value of α_j of fore-slope is 0.73 ± 0.35 and after-slope is 1.16 ± 0.21 .

blogger randomly decides to post a blog including the j -th word. There is a key parameter in this stochastic process; the share of the j -th word c_j is defined by the following equation

$$c_j = \frac{\langle x_j \rangle}{\langle X \rangle}, \quad (\text{B1})$$

where $x_j(t)$ is the number of blog entries including the j -th word on the t -th day. $X(t)$ is the number of active bloggers on the t -th day and the brackets show the mean over all instances. We assume that the number of active bloggers $X(t)$, $X(t) \geq 0$ fluctuates randomly following an independent probability density distribution $\phi(X)$ with finite moments. Probability of posting x_j entries is calculated using a Poisson distribution with the mean number $c_j X$ given as follows

$$P(x_j | c_j) = \int_0^\infty \phi(X) \exp(-c_j X) \frac{(c_j X)^{x_j}}{x_j!} dX. \quad (\text{B2})$$

When $\langle x_j \rangle$ is small, a Poisson distribution is approximated by a Bernoulli distribution that assumes $x_j = 0$ with a probability $1 - c_j X$, and $x_j = 1$ with a probability $c_j X$. Thus, we have the following evaluations for an

arbitrary distribution of $\phi(X)$.

$$\begin{aligned} P(x_j = 0|c_j) &\simeq \int_0^\infty \phi(X) (1 - c_j X) dX \\ &\simeq 1 - c_j \langle X \rangle, \\ P(x_j = 1|c_j) &\simeq \int_0^\infty \phi(X) (c_j X) dX \\ &\simeq c_j \langle X \rangle. \end{aligned} \quad (\text{B3})$$

For $\langle x_j \rangle \approx 2$, $P(x_j \geq 2|c_j) \approx 0$, thereby $P(x_j|c_j)$ is approximated by the Poisson distribution with both the mean and the variance given by $c_j \langle X \rangle$.

For $\langle x_j \rangle \gg 1$, the Poisson distribution can be approximated by a normal distribution,

$$P(x_j|c_j) \simeq \int_0^\infty \phi(X) \frac{1}{\sqrt{2\pi c_j X}} \exp \left[-\frac{(x_j - c_j X)^2}{2c_j X} \right] dX. \quad (\text{B4})$$

By introducing a new variable $y_j = \frac{x_j}{c_j \langle X \rangle}$, Eq. (B4) becomes

$$P(y_j|c_j) \simeq \int_0^\infty \phi(X) \frac{1}{\sqrt{2\pi \left(\frac{X}{c_j \langle X \rangle^2} \right)}} \exp \left[-\frac{(y_j - \frac{X}{\langle X \rangle})^2}{2 \left(\frac{X}{c_j \langle X \rangle^2} \right)} \right] dX. \quad (\text{B5})$$

When $\langle x_j \rangle = c_j \langle X \rangle \gg 1$, the weight function in the integral can be approximated by Dirac's delta function as

$$P(y_j|c_j) \simeq \int_0^\infty \phi(X) \delta \left(y_j - \frac{X}{\langle X \rangle} \right) dX. \quad (\text{B6})$$

Therefore, we have the following simple evaluation, for x_j ,

$$P(x_j|c_j) \simeq \frac{1}{c_j} \phi \left(\frac{x_j}{c_j} \right). \quad (\text{B7})$$

Calculating the first and second moments of $P(x_j|c_j)$, we now have the general results

$$\begin{aligned} \langle x_j \rangle &= \int_0^\infty x_j P(x_j|c_j) dx_j \\ &\simeq \int_0^\infty x_j \frac{1}{c_j} \phi \left(\frac{x_j}{c_j} \right) dx_j = c_j \langle X \rangle, \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} \langle x_j^2 \rangle &= \int_0^\infty x_j^2 P(x_j|c_j) dx_j \\ &\simeq \int_0^\infty x_j^2 \frac{1}{c_j} \phi \left(\frac{x_j}{c_j} \right) dx_j = c_j^2 \langle X^2 \rangle. \end{aligned} \quad (\text{B9})$$

From these results, the standard deviation $\sigma_j = \sqrt{\langle x_j^2 \rangle - \langle x_j \rangle^2}$ can be expressed as

$$\sigma_j \simeq \sqrt{c_j^2 (\langle X^2 \rangle - \langle X \rangle^2)}. \quad (\text{B10})$$

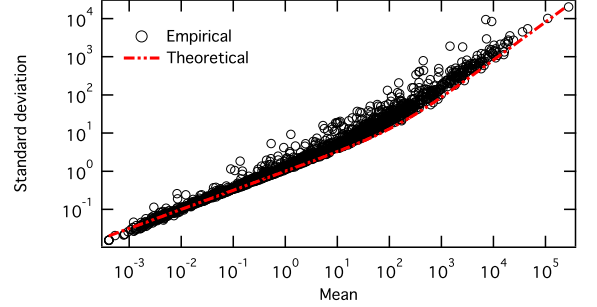


FIG. 11: (Color online) Relationship between mean and standard deviation of word frequency in the blogosphere. Empirical results of 1771 adjectives and theoretical result of Eq. (B12) are duplicated in the figure.

By correlating both results Eqs. (B3) and (B10), we can get the following relation;

$$\sigma_j \simeq \sqrt{c_j \langle X \rangle + c_j^2 \langle X^2 \rangle_c}, \quad (\text{B11})$$

where $\langle X^2 \rangle_c$ denotes the second order cumulant. By using $\langle x_j \rangle = c_j \langle X \rangle$, we rewrite Eq. (B11) into

$$\sigma_j \simeq \sqrt{\langle x_j \rangle + \left(1 + \frac{\langle X^2 \rangle_c}{\langle X \rangle^2} \langle x_j \rangle \right)}. \quad (\text{B12})$$

Figure 11 shows empirical results using 1771 adjectives and Eq. (B12) with $\frac{\langle X^2 \rangle_c}{\langle X \rangle^2} = 0.08$.

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